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When and why do you multiply probabilities?

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At high school you may have been taught that, sometimes, you have to multiply probabilities. We briefly discuss when and why you do this.

First a few notes on the notation of probabilities. When throwing with a dice, the event of throwing a six is 1 of 6 possibilities. We write this as a fraction, 1/6, or

$$\frac{1}{6}.$$

We then say there is a probability of 1 out of 6 to throw, for instance, a 6. The probability is 1/6, one sixth.

This also means that the probability of throwing *a* number—this can thus be *any* number: 1, 2, 3, 4, 5 or 6—is equal to

$$\frac{6}{6} = 1.$$

If you throw a dice, the probability is 1 for throwing *a* number, or 100%. In other words, if something is 100% certain to happen, the probability is 1. And if something is less certain to occur, less than 100%, the probability is an n'th part of 1.

Lastly, an important announcement on multiplying by a fraction: if you calculate an n'th part of something, for instance, 16, you can write this in two ways. You divide 16 by 2 or you multiply 16 by 1/2. It is the same. That is:

$$\frac{16}{2} = 16 \times \frac{1}{2} = 8.$$

Two coins

Suppose, you throw euro #1 into the air. It is going to be either heads or tails. In Figure 1, this is represented schematically. The probability of throwing heads is 1/2. The odds of throwing tails is 1/2.

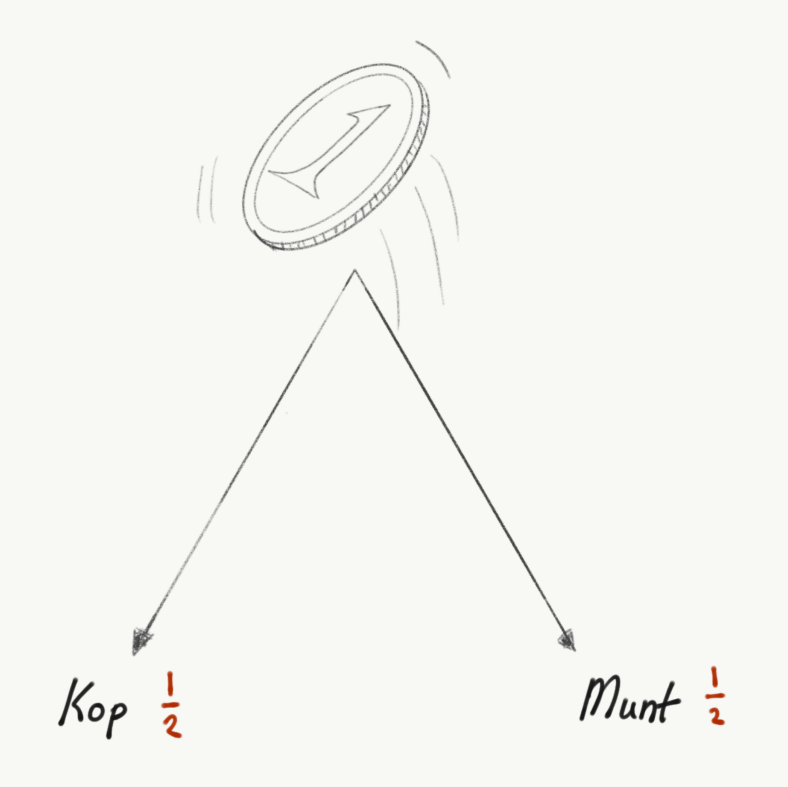

Figure 1: Heads or tails

Imagine throwing euro #1 and euro #2 into the air. This is represented in Figure 2.

Now, ask yourself the question: of all the times I threw heads with euro #1, how many times would I have thrown euro #2? The answer is that half of the time euro #1 became heads and half of *that* time europ #2 became heads.

What is half of a half? This is

$$\underbrace{\frac{1/2}{2}}_{\text{half of a half}} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

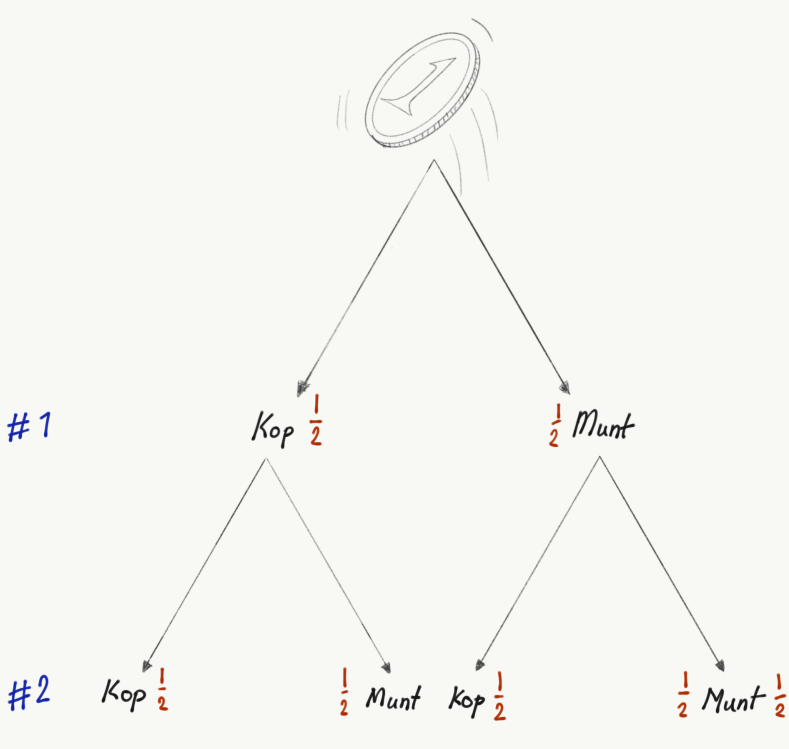

Figure 2: Heads or tails

Part of a part

See Figure 3. Suppose, we throw two coins 16 times. Suppose, coin number 1 turns out heads half the time; we signify this with blue circles. The question is how many times that coin number 1 is heads, do we throw heads with the second coin? This is, again, half. Half of half, that is. We paint this green.

Of the total amount of throws, what part is green? Half (4) of half (8) of the total (16), so 4 out of 16, or 1 out of 4. So, what is the probability of throwing green (coin number 2 is heads) if you throw blauw (coin number 1 is heads) half of the time.

$$\frac{1/2}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

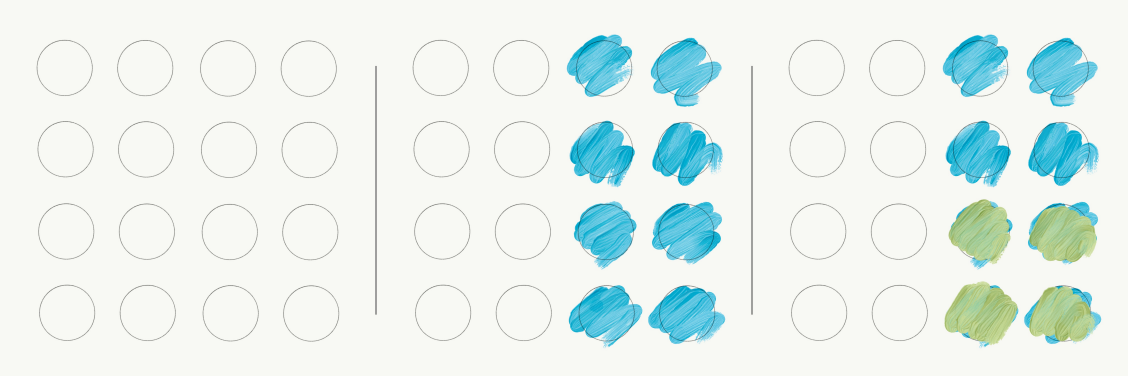

Figure 3: From left to right: the total amount of throws, half of the total, and half of that half

A euro and a dice

Another example. Suppose, you throw a euro and a dice into the air. The probability distribution of heads and tails is 1/2, as we know. In the case of the dice this is different: it can turn out to be 1, 2, 3, 4, 5 or 6. So, the probability of throwing a six is 1/6.

Of all the trials where the euro turned out to be heads—which is half of the total amount of trials—how many times would you have thrown a 6 with the dice? That is, thus, *1/6th of one half* of the total amount of trials. Or,

$$\frac{1/2}{6} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}.$$

Conclusion

Suppose, that the probability is 2/3 for event A to happen, the probability is 1/6 for event B to occur, and 4/5 that even C will happen, then the probability of the combination of the events A, B, and C to occur is equal to

$$\underset{A}{\frac{2}{3}} \times \underset{B}{\frac{1}{6}} \times \underset{C}{\frac{4}{5}} = \frac{8}{90} = \frac{4}{45} \approx 0.088\ldots,$$

which is 8.9% rounded to one decimal.

To know what the probability of a combination of events occurring is, we calculate the n'th time of an n'th time. And the n'th time of an n'th time (of an n'th time, etc...) is the same als multiplying the two (or more) fractions.